The Mathematics of Democracy:
Political Ideals and the Theory of Voting

Winter, 20??

Instructor: Gerard Rothfus
Classroom: Online
Day/Time: W: 12-1:50pm
Office Hours: M, W, F: 11-11:50am or by appointment
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Description

In a democracy, we want elections to be fair and outcomes to reflect the will of the voters. But what exactly does it mean for an election to be fair or for its results to reflect the voters' will? This course examines these questions through the lens of the mathematical theory of voting, uncovering an array of formal puzzles and philosophical quandaries along the way. Topics covered include comparison of popular voting methods (e.g. plurality vote, ranked choice, Borda count, etc.), famous voting paradoxes (incl. the Condorcet Paradox and Sen's Liberal Paradox), Arrow's Impossibility Theorem, strategic voting (incl. the Gibbard-Satterthwaite Theorem), and the mathematics of apportionment.

Learning Objectives

This course will equip students to:

- Recognize the virtues and vices of a wide array of commonly used voting methods.

- Understand the content and significance of the central results of axiomatic voting theory, especially Arrow's Impossibility Theorem.

- Reflect thoughtfully about both the meaning of widely shared democratic ideals and how to best realize them in our electoral procedures.

- Gain a deeper appreciation of contemporary debates surrounding apportionment in the U.S. political system.

Course Materials

There are two required texts for the course and one optional text:

- *Liberalism Against Populism: A Confrontation Between the Theory of Democracy and the Theory of Social Choice* by William Riker, 1982, Waveland.

- *Collective Choice and Social Welfare* (1st or 2nd edition) by Amartya Sen, 1970/2016, Harvard.

- **Optional:** *Fair Representation: Meeting the Ideal of One Man, One Vote* by Michel Balinski and H. Peyton Young, 2001, Brookings.

The Balinski/Young text on apportionment is optional since I will also make available class notes on this topic and there will be no homework problems drawn from the text.

Course Structure

This course will be organized around a weekly lecture/discussion period, where various topics in social choice theory will be explored and discussed in person. You are encouraged to do the suggested readings before each lecture in order to be better prepared to engage and ask questions, make suggestions in discussion, etc. Periodically, I will also release both lecture notes and short, pre-recorded videos to the ILIAS site (as well as YouTube), summarizing different ideas covered in the course. You may review these notes and videos to help with homework problems or just to get a better grip on key ideas in the course.

I will also hold office hours at various times during the week. You are encouraged to attend any of these office hours that you like! During these, I will go over questions regarding homework assignments and may (anonymously) review students' submitted answers. You are also encouraged to come for the purpose of asking any questions you may have about the course! Please feel truly free to reach out to me at any time.

Homework and Exams

Homework will be due at the beginning of class every three weeks (Wednesday at noon) and should be submitted via ILIAS. Late homework will not be accepted without a medical or family emergency, though your lowest homework grade will be dropped, so you can miss one of the four homework assignments without penalty. There will also be a cumulative, in class final exam. Feel free to use any notes or books during the homeworks, but the final exam will be closed notes/books. You may also discuss homework problems together (in fact

working together is highly encouraged!), though every student must write/type out their own exam/homework.

Election Project

One theme of this course will be that, aside from the preferences of the voters, the choice of an *election method* can have a decisive influence on election outcomes. To give ourselves the opportunity to bring the significance this point home in a vivid way, our final project for the course will involve finding historical examples in which this phenomenon rears its head. The details of this project can be found in a separate document uploaded to CANVAS.

In short, you will be asked to work with two partners (randomly assigned by myself in the third week of class) on crafting a 20 minute in-class presentation making the case that a particular real life election would have turned out differently if it had employed one of the different voting methods surveyed in this course. The election can be a political election from any country/state or a non-political election (e.g. a sporting or talent competition involving voting judges, etc.). Your presentation should address at least two of the voting methods discussed in this class and why the outcome of the election likely would have varied depending on the choice of method. Your group must run your proposed election topic by me before Jan 12, or three before the presentations, for approval. You will be graded on completing the assignment, working well as a team, explaining voting methods accurately and clearly, and effectively arguing for your claims.

Grading

- Final Exam: 30%
- Election Project: 30%
- Homework: 30%
- Participation: 10%

Grade Scale

A: 90-100	B: 80-89	C: 70-79	D: 60-69	F: < 60

Academic Integrity

Academic dishonesty will not be tolerated. The UCI Academic Integrity Policy will be followed in this course, and it is the resposnibility of the student to adhere

to these policies: https://aisc.uci.edu/students/academic-integrity/index.php. Students who have any questions or uncertainty about this policy are responsible for meeting with the instructor to discuss the policy.

Disabilities

Please notify me in advance of the need for accommodation of a University verified disability. I will gladly provide the required accommodations. If you have any questions or concerns about disability accommodations, please don't hesitate to speak with me; I am happy to help out.

Course Outline

Week:	Topic:
Oct 27	**Democracy and Voting** • What is Democracy?, Is Plurality Vote Democratic? The Importance of Election Methods and Voting Theory **Reading:** • *Liberalism Against Populism* by William Riker: Chapter 1, "The Connection between the Theory of Social Choice and the Theory of Democracy" and Chapter 2, "Different Choices from Identical Values". **Homework One Due Nov 17**
Nov 3	**Logic and Set Theory: The Language of Voting Theory** • Finite Set Theory, Preference Relations, Choice Functions **Reading:** • *Collective Choice and Social Welfare* by Amartya Sen: Chapter 1*, "Preference Relations".
Nov 10	**Majority Rule** • Simple Majority Rule, May's Theorem, Condorcet's Jury Theorem **Reading:** • *Collective Choice and Social Welfare* by Amartya Sen: Chapter 5*, "Anonymity, Neutrality, and Responsiveness". • *Liberalism Against Populism* by William Riker: Chapter 3, "Simple Majority Decision".
Nov 17	**The Condorcet Paradox** • The Condorcet Paradox, Pairwise Voting, Single-Peakedness **Reading:** • *Collective Choice and Social Welfare* by Amartya Sen: Chapter 10, "The Method of Majority Decisions". **Homework Two Due Dec 8**
Nov 24	**Voting Methods I** • Majoritarian Methods, Multi-round Voting, Alternative Vote **Reading:** • *Liberalism Against Populism* by William Riker: Chapter 4.A-C.

Week:	Topic:
Dec 1	**Voting Methods II** • Positional Methods, Approval Voting, Borda Count **Reading:** • *Liberalism Against Populism* by William Riker: Chapter 4.D-E.
Dec 8	**Voting Methods III** • Utilitarian Voting Methods; Measurement of Cardinal Utility **Reading:** • *Liberalism Against Populism* by William Riker: Chapter 4.F-G. • *Collective Choice and Social Welfare* by Amartya Sen: Chapter 7, "Interpersonal Aggregation and Comparability". **Homework Three Due Jan 12**
Dec 15	**Arrow's Impossibility Theorem** • Pareto Optimality, Independence of Irrelvant Alternatives, Proof and Significance of Arrow's Theorem **Reading:** • *Collective Choice and Social Welfare* by Amartya Sen: Chapter 3/3*, "Collective Rationality/Social Welfare Functions". • *Liberalism Against Populism* by William Riker: Chapter 5, "The Meaning of Social Choice".
Dec 22	**Liberalism and Democracy** • Sen's Liberal Paradox, its Proof and Significance **Reading:** • *Collective Choice and Social Welfare* by Amartya Sen: Chapter 6/6*, "Conflicts and Dilemmas/The Liberal Paradox".
Jan 12	**Strategic Voting** • Manipulation of Voting Systems, the Gibbard-Satterthwaite Theorem **Reading:** • *Liberalism Against Populism* by William Riker: Chapter 6, "The Manipulation of Social Choice: Strategic Voting". **Homework Four Due Feb 2**

Week:	Topic:
Jan 19	**Apportionment I** • Apportionment in the United States, Historical Views, Paradoxes **Reading:** • *Fair Representation* by Balinski and Young: Chapters 1-4.
Jan 26	**Apportionment II** • Apportionment Methods, Bias, Staying within the Quota **Reading:** • *Fair Representation* by Balinski and Young: Chapters 6-12. **Homework Three Due Jan 12**
Feb 2	**Project Presentations I**
Feb 9	**Project Presentations II**
Feb 16	**Final Exam**